U.S. Forest Service
Forest Inventory and Analysis

Uncertainty in Volume Estimates for Stands Reconstructed from Stump Dimensions

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Introduction

Stand Reconstruction

- Often the purpose of reconstructing a stand is to determine the amount and value of removed volume.
- Suspicion: Generated volumes and associated value are treated as a known quantity with no (or at least unquantified) uncertainty.
- Outcome: Uncertainty is not taken into account in subsequent use of these estimates.

Introduction

Stand Reconstruction Uncertainty

- Sampling error (if any)
  - Stumps
  - Plots

- Model error
  - Residual
  - Parameter estimates

- Stump measurement error – not considered here
  - Species
  - Diameter
  - Height

Introduction

- Objectives

  1) Compare model-related uncertainty amongst various models used in the prediction of tree volume based on stump data.

  2) Evaluate the relative contributions of sampling error and model-related uncertainty for estimates of stand volume and associated economic value over a range of sample intensities.

- Data

  1) NE Taper data (Westfall and Scott 2010) – Model development (18 species/groups) to quantify residual and parameter error.

  2) Taper data NFS (Allegheny, Green Mountain, White Mountain) lowest measurement (~ 1 ft) used to mimic stumps of a cut stand : N = 898 (legacytreedata.org)

Approaches

1) Predict dbh from stump information; predict volume from dbh only
2) Predict dbh from stump information; predict height from dbh; predict volume from dbh and height
3) Predict volume directly from stump diameter and height

Methods

Models

Dbh

$$dbh = d \times \left(1.37 / h\right)^{\beta_1} + \beta_2\left(1.37 - h\right) + \varepsilon$$

$$\varepsilon \sim N\left(0, \beta_3 d^{\beta_4}\right)$$

Merchantable height

$$HT_m = \theta_1(1 - \exp(\theta_2 dbh))^{\theta_3} + \varepsilon$$

$$\varepsilon \sim N\left(0, HT_m^{\theta_4}\right)$$

Methods

Models

Volume from dbh

$$V_d = \delta_1 + \delta_2 dbh^2 + \varepsilon$$

$$\varepsilon \sim N\left(0, dbh^{\delta_3}\right)$$

Volume from dbh and height

$$V_{dh} = \lambda_1 + \lambda_2 \left(dbh^2 HT_m\right)^{\lambda_3} + \varepsilon$$

$$\varepsilon \sim N\left(0, dbh^{\lambda_4 HT_m}\right)$$

Methods

Models

Volume from stump diameter and height

$$V_{st} = \gamma_1 d^{\gamma_2} h^{\gamma_3} + \varepsilon$$

$$\varepsilon \sim N\left(0, \gamma_4 d^{\gamma_5}\right)$$

Methods

Model Uncertainty

- Parameter uncertainty
  - 5000 bootstrap samples – refit model and save parameter estimates – distribution and covariance
- Residual error
  - Select random z from N(0,1)
  - RE = z x sqrt(Var(ε))
- Predict new observation
  - Choose random set of parameters – obtain prediction
  - Add the RE to the prediction

Methods

- Independent variable error propagation
  - Dbh and merchantable height variation passed into respective volume models

$$\hat{V}_{dh} = \hat{\lambda}_1 + \hat{\lambda}_2\left(\widehat{dbh}^2\,\widehat{HT}_m\right)^{\hat{\lambda}_3} + z\sqrt{\widehat{dbh}^{\hat{\lambda}_4 \widehat{HT}_m}}$$

- Repeat 5000 times for each type of volume estimate to obtain distribution of population estimates of total volume

- Convert cubic volume to board feet: 158.9 BF = 1 m^3

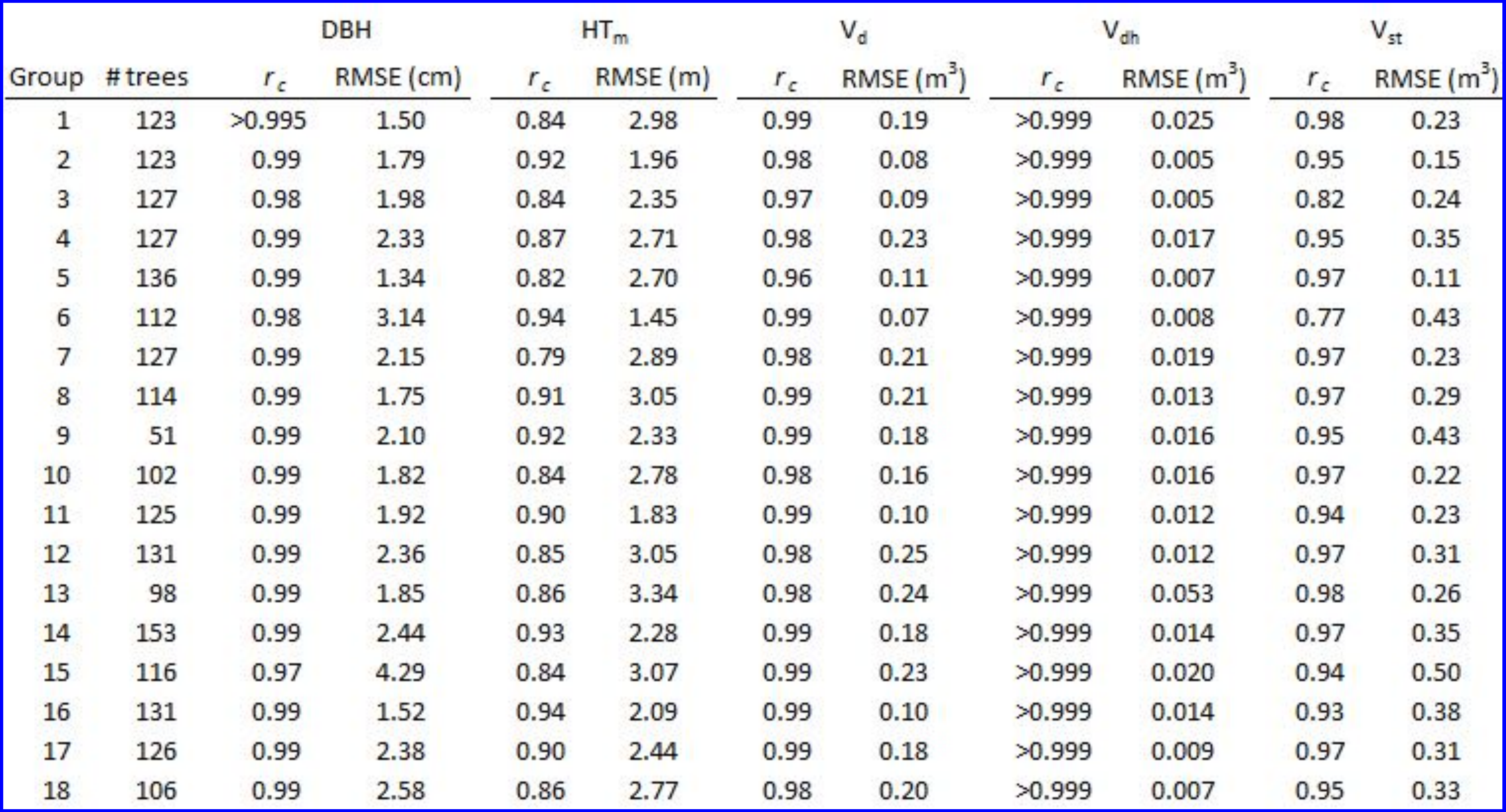
- Convert BF to \$US: \$100 softwood, \$250 hardwood per MBF

Methods

Sampling Design

- Proportions: 0.05, 0.10,…0.50, 1.00
- Stumps – SRS (N = 898)
- Plots – SRS
  - Plot size 0.042 ha (1/10 acre)
  - N = 110
  - Stumps randomly assigned to a plot prior to analysis (average of ~ 8 stumps per plot)

Results

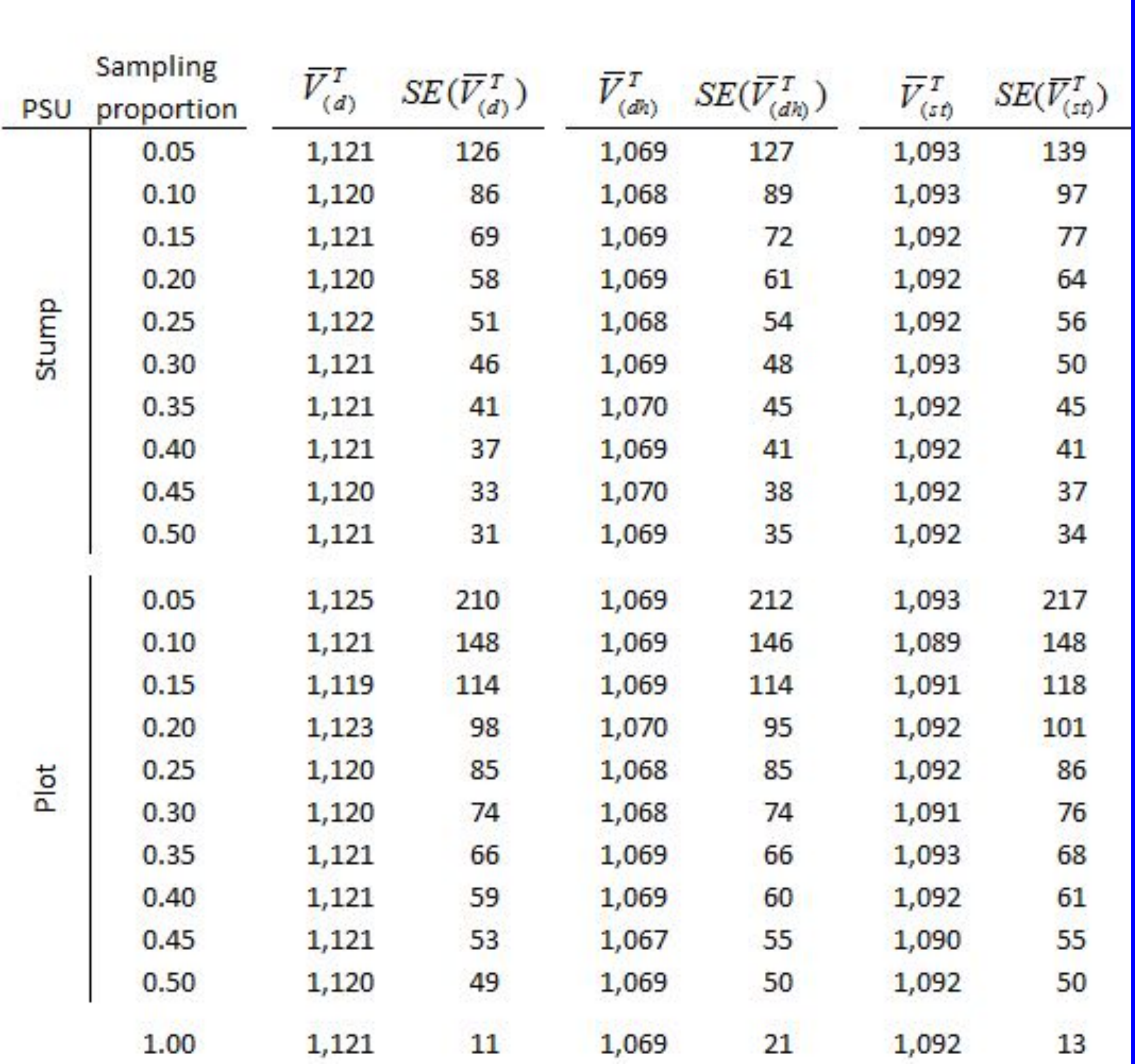


		DBH		HT_m		V_d		V_{dh}		V_{st}	
Group	# trees	r_c	RMSE (cm)	r_c	RMSE (m)	r_c	RMSE (m^3)	r_c	RMSE (m^3)	r_c	RMSE (m^3)
1	123	>0.995	1.50	0.84	2.98	0.99	0.19	>0.999	0.025	0.98	0.23
2	123	0.99	1.79	0.92	1.96	0.98	0.08	>0.999	0.005	0.95	0.15
3	127	0.98	1.98	0.84	2.35	0.97	0.09	>0.999	0.005	0.82	0.24
4	127	0.99	2.33	0.87	2.71	0.98	0.23	>0.999	0.017	0.95	0.35
5	136	0.99	1.34	0.82	2.70	0.96	0.11	>0.999	0.007	0.97	0.11
6	112	0.98	3.14	0.94	1.45	0.99	0.07	>0.999	0.008	0.77	0.43
7	127	0.99	2.15	0.79	2.89	0.98	0.21	>0.999	0.019	0.97	0.23
8	114	0.99	1.75	0.91	3.05	0.99	0.21	>0.999	0.013	0.97	0.29
9	51	0.99	2.10	0.92	2.33	0.99	0.18	>0.999	0.016	0.95	0.43
10	102	0.99	1.82	0.84	2.78	0.98	0.16	>0.999	0.016	0.97	0.22
11	125	0.99	1.92	0.90	1.83	0.99	0.10	>0.999	0.012	0.94	0.23
12	131	0.99	2.36	0.85	3.05	0.98	0.25	>0.999	0.012	0.97	0.31
13	98	0.99	1.85	0.86	3.34	0.98	0.24	>0.999	0.053	0.98	0.26
14	153	0.99	2.44	0.93	2.28	0.99	0.18	>0.999	0.014	0.97	0.35
15	116	0.97	4.29	0.84	3.07	0.99	0.23	>0.999	0.020	0.94	0.50
16	131	0.99	1.52	0.94	2.09	0.99	0.10	>0.999	0.014	0.93	0.38
17	126	0.99	2.38	0.90	2.44	0.99	0.18	>0.999	0.009	0.97	0.31
18	106	0.99	2.58	0.86	2.77	0.98	0.20	>0.999	0.007	0.95	0.33

Results

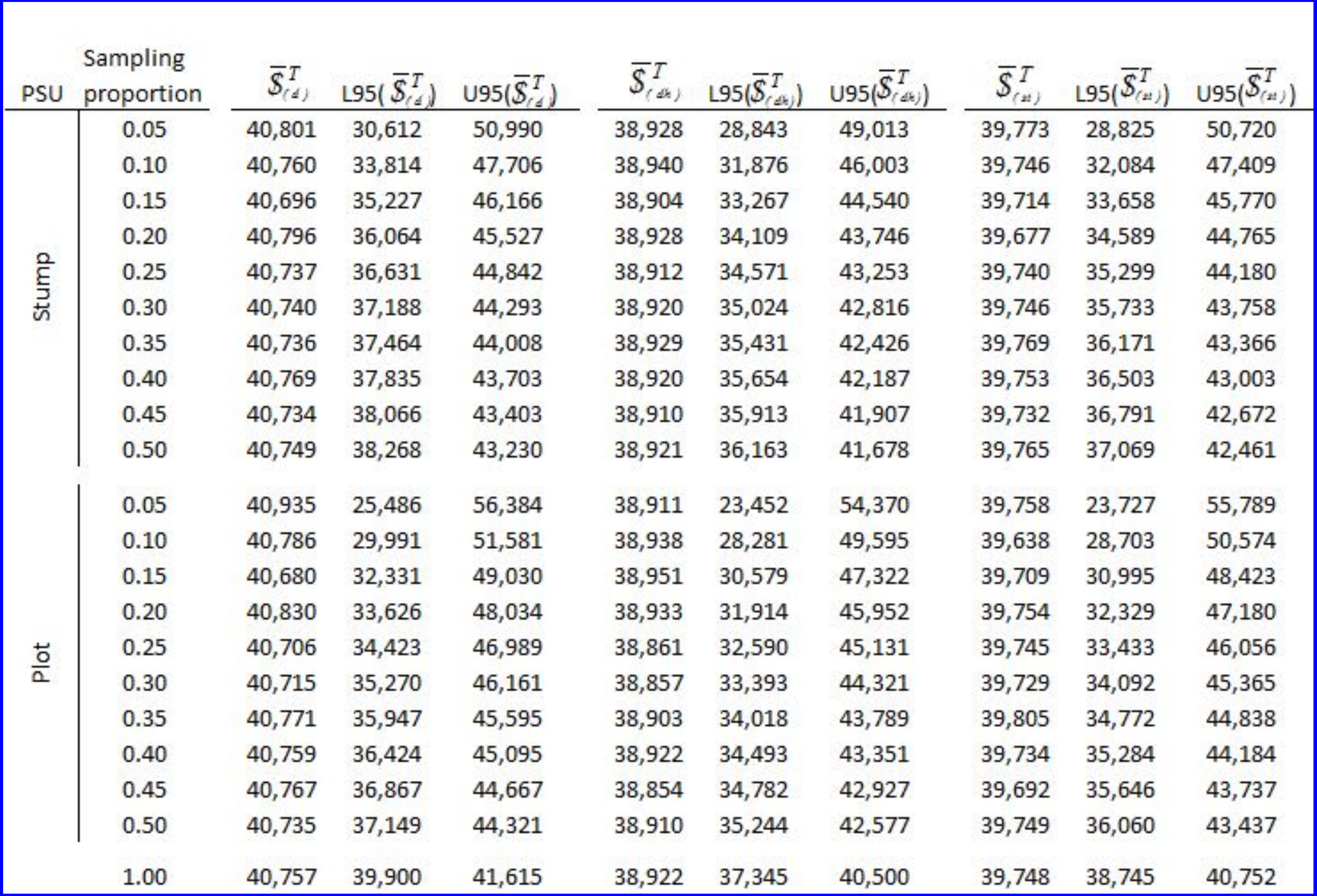
- Complete enumeration (model error only)
- V_d : 0.68% dbh + 0.32% volume = 1.0% Std Err
- V_{dh} : 0.59% dbh + 1.23% height + 0.08% volume = 1.9% Std Err
- V_{st} : 1.2% volume = 1.2% Std Err
- Preferred order – 1)V_d, 2) V_{st}, 3) V_{dh}

Results



PSU	Sampling proportion	$\bar{V}^T_{(d)}$	$SE(\bar{V}^T_{(d)})$	$\bar{V}^T_{(dh)}$	$SE(\bar{V}^T_{(dh)})$	$\bar{V}^T_{(st)}$	$SE(\bar{V}^T_{(st)})$
Stump	0.05	1,121	126	1,069	127	1,093	139
	0.10	1,120	86	1,068	89	1,093	97
	0.15	1,121	69	1,069	72	1,092	77
	0.20	1,120	58	1,069	61	1,092	64
	0.25	1,122	51	1,068	54	1,092	56
	0.30	1,121	46	1,069	48	1,093	50
	0.35	1,121	41	1,070	45	1,092	45
	0.40	1,121	37	1,069	41	1,092	41
	0.45	1,120	33	1,070	38	1,092	37
	0.50	1,121	31	1,069	35	1,092	34
Plot	0.05	1,125	210	1,069	212	1,093	217
	0.10	1,121	148	1,069	146	1,089	148
	0.15	1,119	114	1,069	114	1,091	118
	0.20	1,123	98	1,070	95	1,092	101
	0.25	1,120	85	1,068	85	1,092	86
	0.30	1,120	74	1,068	74	1,091	76
	0.35	1,121	66	1,069	66	1,093	68
	0.40	1,121	59	1,069	60	1,092	61
	0.45	1,121	53	1,067	55	1,090	55
	0.50	1,120	49	1,069	50	1,092	50
	1.00	1,121	11	1,069	21	1,092	13

Results



PSU	Sampling proportion	$\overline{\mathbb{S}}^T_{(d)}$	L95($\overline{\mathbb{S}}^T_{(d)}$)	U95($\overline{\mathbb{S}}^T_{(d)}$)	$\overline{\mathbb{S}}^T_{(dk)}$	L95($\overline{\mathbb{S}}^T_{(dk)}$)	U95($\overline{\mathbb{S}}^T_{(dk)}$)	$\overline{\mathbb{S}}^T_{(st)}$	L95($\overline{\mathbb{S}}^T_{(st)}$)	U95($\overline{\mathbb{S}}^T_{(st)}$)
Stump	0.05	40,801	30,612	50,990	38,928	28,843	49,013	39,773	28,825	50,720
	0.10	40,760	33,814	47,706	38,940	31,876	46,003	39,746	32,084	47,409
	0.15	40,696	35,227	46,166	38,904	33,267	44,540	39,714	33,658	45,770
	0.20	40,796	36,064	45,527	38,928	34,109	43,746	39,677	34,589	44,765
	0.25	40,737	36,631	44,842	38,912	34,571	43,253	39,740	35,299	44,180
	0.30	40,740	37,188	44,293	38,920	35,024	42,816	39,746	35,733	43,758
	0.35	40,736	37,464	44,008	38,929	35,431	42,426	39,769	36,171	43,366
	0.40	40,769	37,835	43,703	38,920	35,654	42,187	39,753	36,503	43,003
	0.45	40,734	38,066	43,403	38,910	35,913	41,907	39,732	36,791	42,672
	0.50	40,749	38,268	43,230	38,921	36,163	41,678	39,765	37,069	42,461
Plot	0.05	40,935	25,486	56,384	38,911	23,452	54,370	39,758	23,727	55,789
	0.10	40,786	29,991	51,581	38,938	28,281	49,595	39,638	28,703	50,574
	0.15	40,680	32,331	49,030	38,951	30,579	47,322	39,709	30,995	48,423
	0.20	40,830	33,626	48,034	38,933	31,914	45,952	39,754	32,329	47,180
	0.25	40,706	34,423	46,989	38,861	32,590	45,131	39,745	33,433	46,056
	0.30	40,715	35,270	46,161	38,857	33,393	44,321	39,729	34,092	45,365
	0.35	40,771	35,947	45,595	38,903	34,018	43,789	39,805	34,772	44,838
	0.40	40,759	36,424	45,095	38,922	34,493	43,351	39,734	35,284	44,184
	0.45	40,767	36,867	44,667	38,854	34,782	42,927	39,692	35,646	43,737
	0.50	40,735	37,149	44,321	38,910	35,244	42,577	39,749	36,060	43,437
	1.00	40,757	39,900	41,615	38,922	37,345	40,500	39,748	38,745	40,752

Conclusion

- Model error relatively small (1 – 2% of estimate), but should still be accounted for.
- Primarily large tree stands may exhibit more model uncertainty (small trees = less).
- Less uncertainty with stumps vs plots (but costs could be higher).
- Small sampling proportion = large uncertainty in valuation – possible use/interpretation of this information??