Prediction of Tree Diameter Growth

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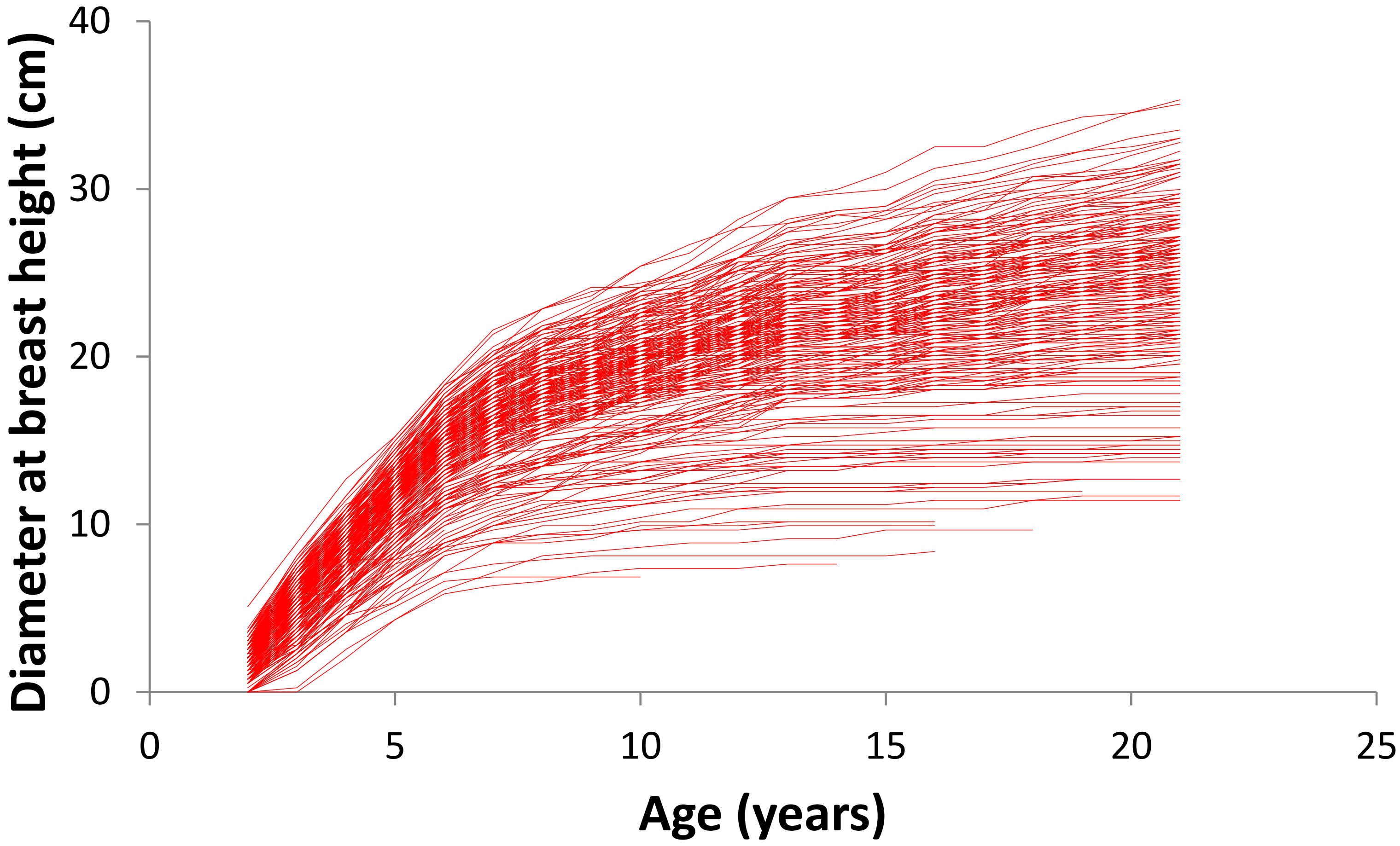
Data

- 340 loblolly pine trees
- 6147 observations
- Planted near Bogalusa, LA
- 9’ x 12’ spacing
- DBH measured annually from age 2 to age 21

Data



Diameter at breast height (cm)
40
30
20
10
0
0
5
10
15
20
25
Age (years)

Model

Bailey and Clutter (1974)

$$y(t_{ij}) = \exp\left(b_1 + b_2 {t_i}^{b_3} + \epsilon_{ij}\right)$$

Mixed-Effects Model

$$y(t_{ij}) = \exp\left[(b_1 + u_1)b_1 + (b_2 + u_2)t_i{}^{b_3}\right] + \epsilon_{ij}$$

Mixed-Effects Model

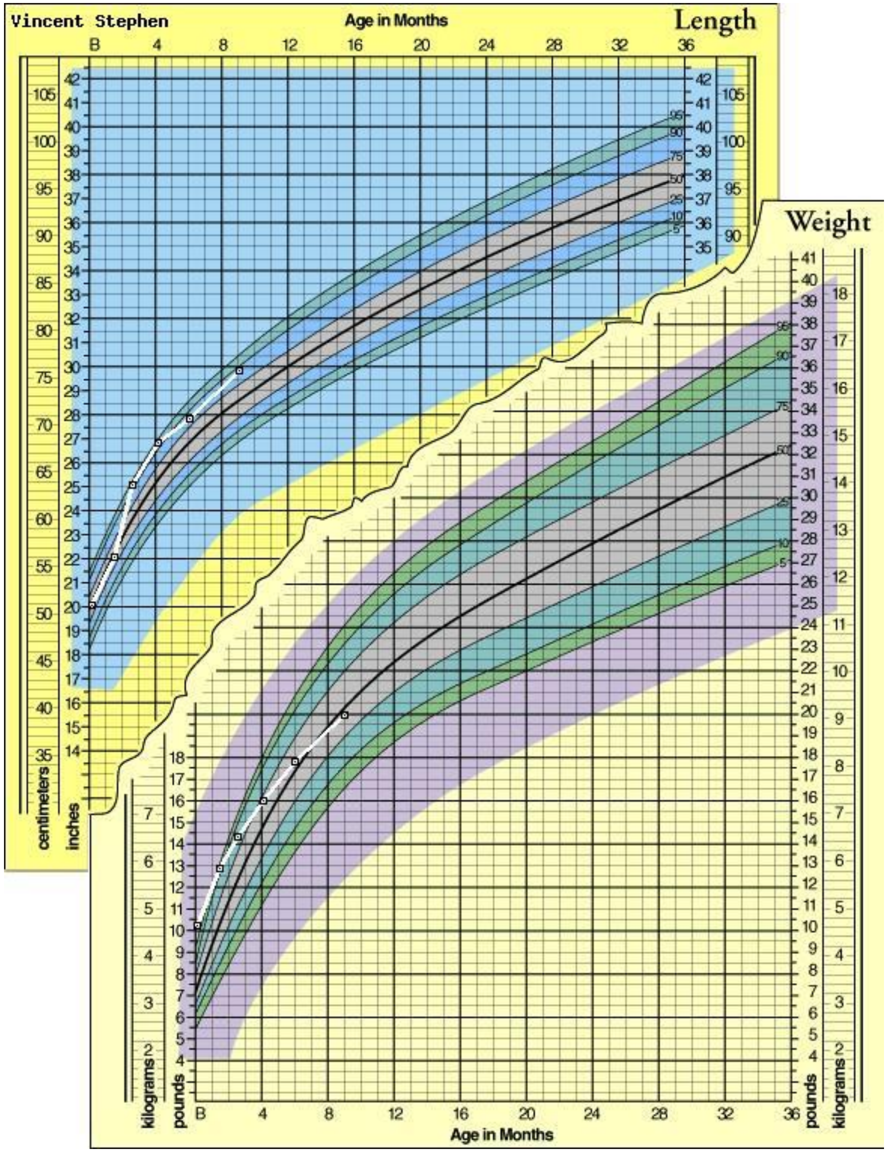
$$y(t_{ij}) = \exp\left[(b_1 + u_1)b_1 + (b_2 + u_2){t_i}^{b_3}\right] + \epsilon_{ij}$$

$$\mathbf{y}_i = \mathbf{f}(\mathbf{b}, \mathbf{u}_i, \mathbf{t}_i) + \boldsymbol{\varepsilon}_i$$

$\boldsymbol{\varepsilon}_i \sim N(\mathbf{0}, \boldsymbol{R})$ and $\boldsymbol{u}_i \sim N(\mathbf{0}, \boldsymbol{D})$

$$\hat{\mathbf{u}}_i^{\,k+1} = \hat{\mathbf{D}}\mathbf{Z}_i^{\mathrm{T}}\left(\mathbf{Z}_i\hat{\mathbf{D}}\mathbf{Z}_i^{\mathrm{T}} + \hat{\mathbf{R}}\right)^{-\mathbf{1}}\left[\mathbf{y}_i - \mathbf{f}\left(\hat{\mathbf{b}}, \hat{\mathbf{u}}_i^{\,k}, \mathbf{t}_i\right) + \mathbf{Z}_i\hat{\mathbf{u}}_i^{\,k}\right]$$

Quantile Regression



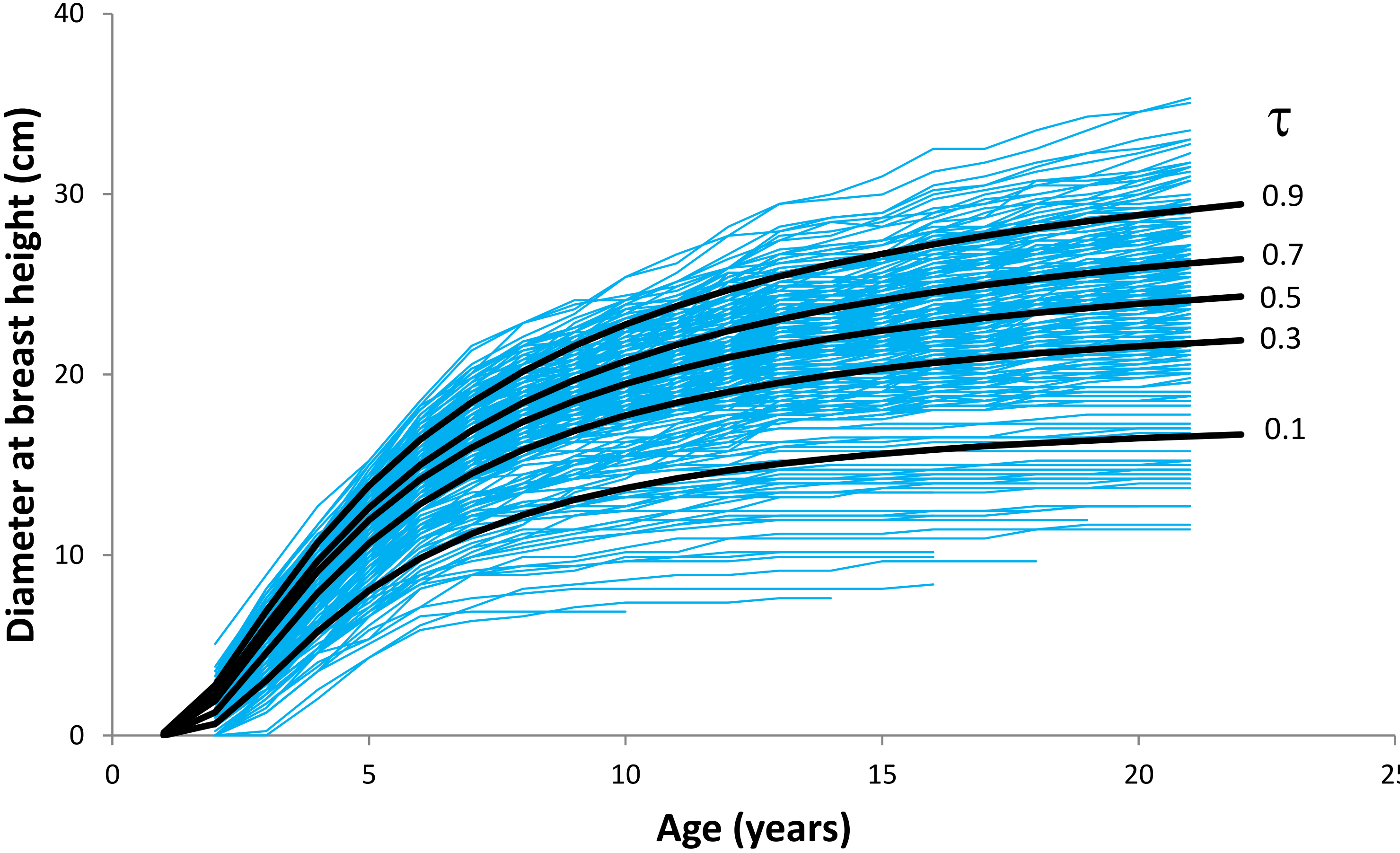
Quantile Regression

$$\hat{y}_i = f(x_i)$$

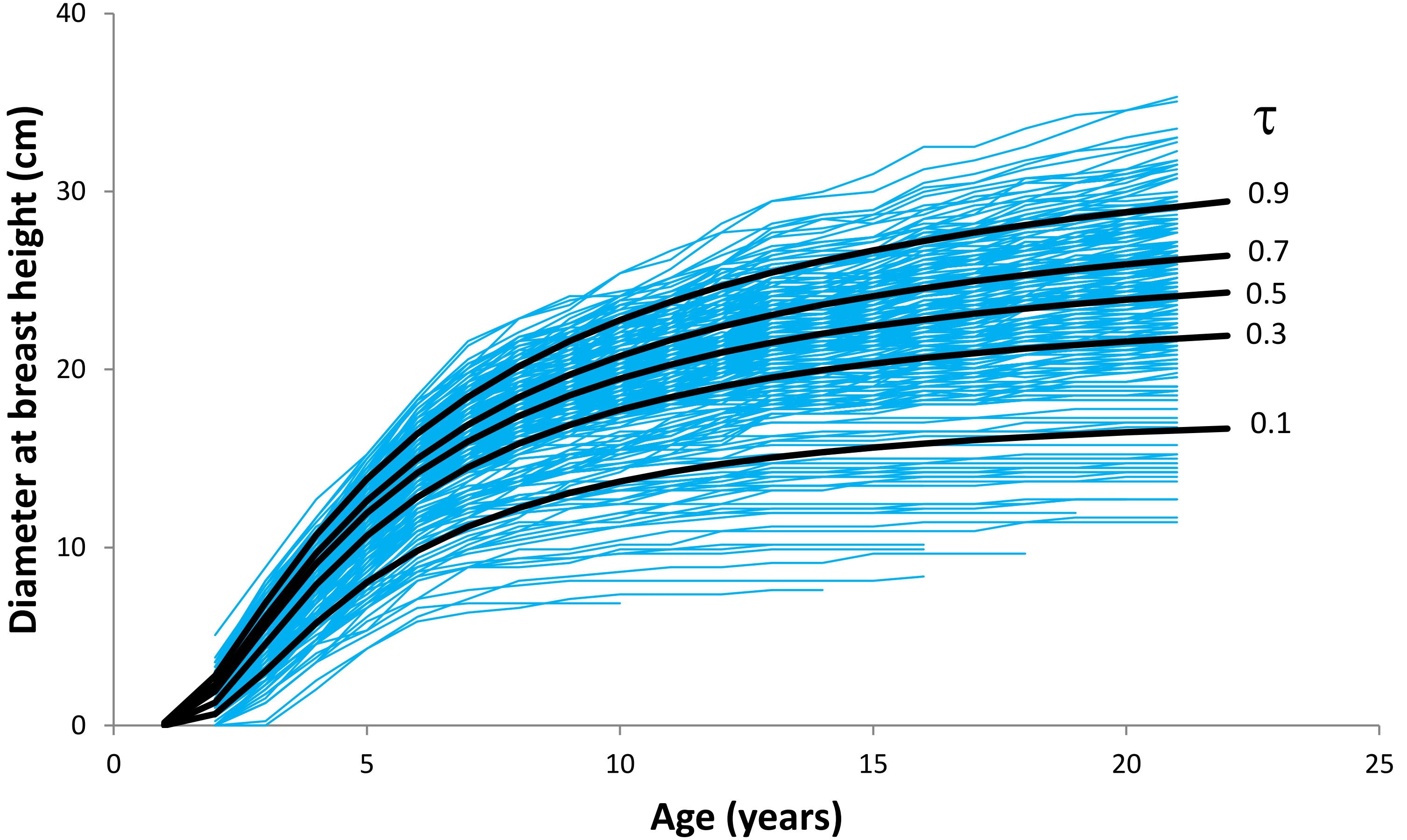
Parameters of $f(x)$ are obtained by minimizing

$$S = \sum_{y_i \geq \hat{y}_i} \tau |y_i - \hat{y}_i| + \sum_{y_i < \hat{y}_i} (1 - \tau) |y_i - \hat{y}_i|$$

Quantile Regression



Quantile Regression



$$S = \sum_{y_i \geq \hat{y}_i} \tau |y_i - \hat{y}_i| + \sum_{y_i < \hat{y}_i} (1-\tau) |y_i - \hat{y}_i|$$

Evaluation

- Models considered
  - Mixed-effects model
  - QR9 (.1, .2, .3, .4, .5, .6, .7, .8, .9)
  - QR5 (.1, .3, .5, .7, .9)
  - QR3 (.1, .5, .9)

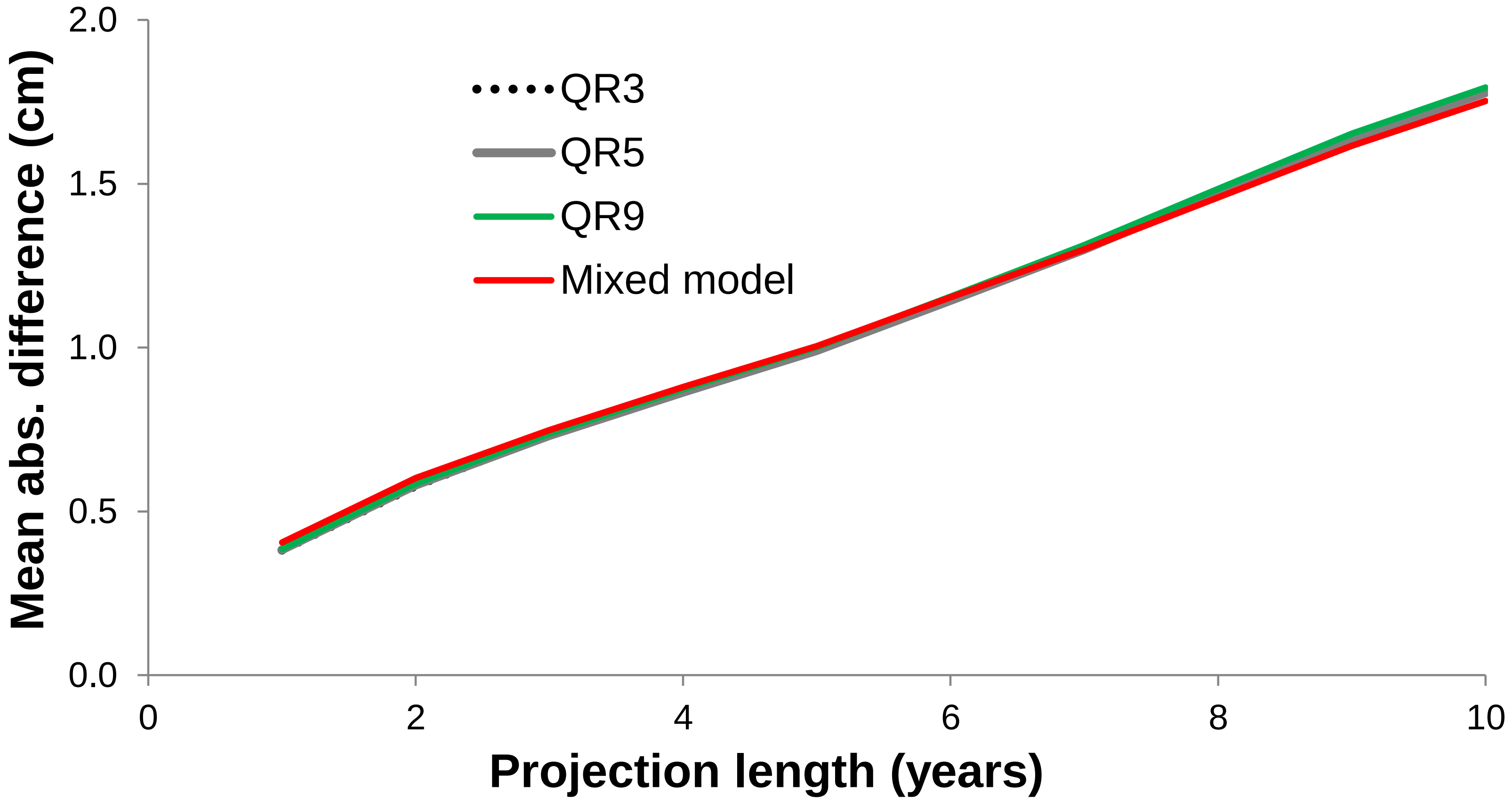
Evaluation

- Models considered
- 5 groups of 68 trees each
- Leave-one-out scheme

Evaluation

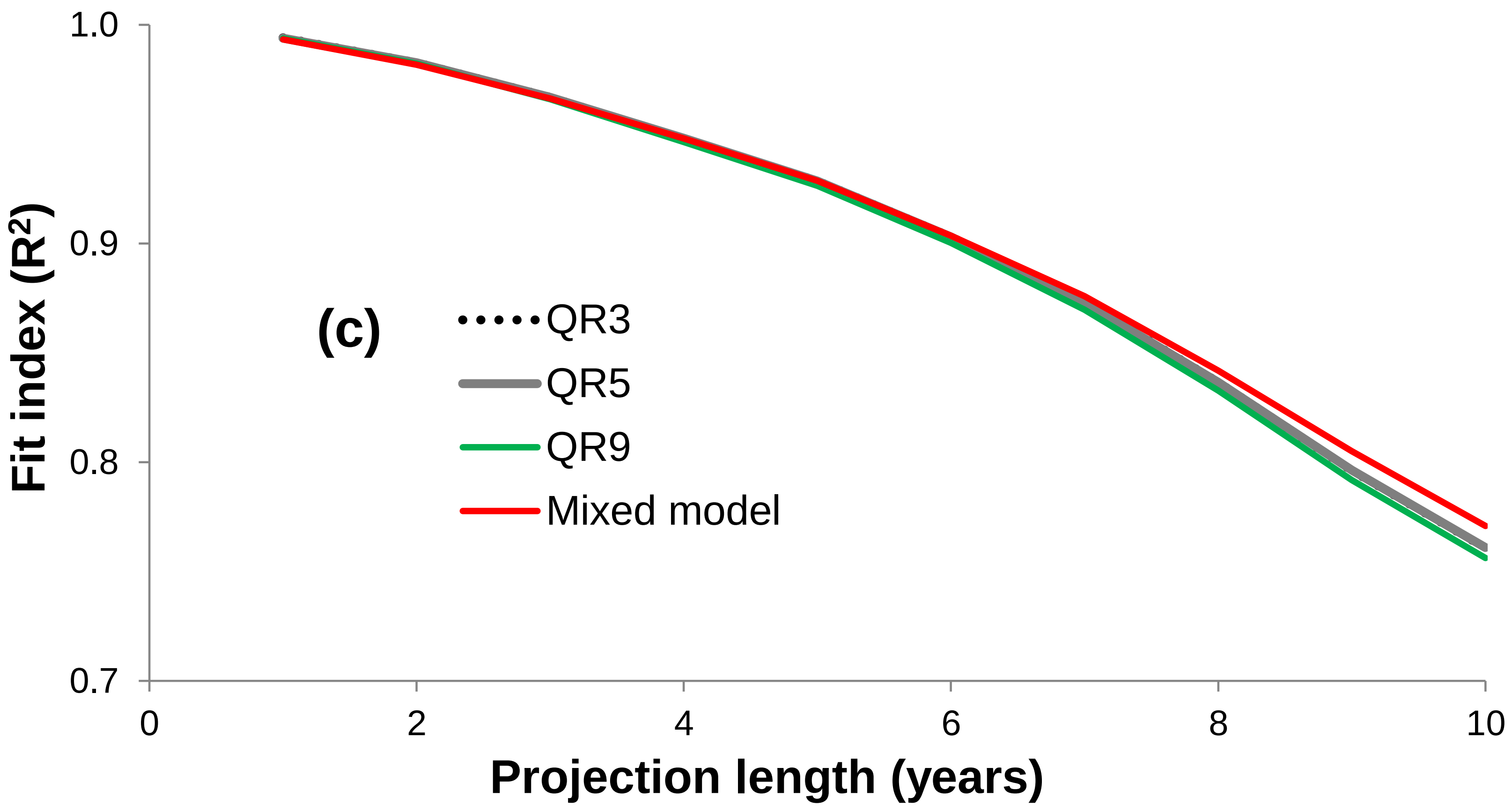
- Models considered
- 5 groups of 68 trees each
- Leave-one-out scheme
- Evaluation statistics
  - MD: Mean difference
  - MAD: Mean absolute difference
  - R^2: Fit index

Evaluation



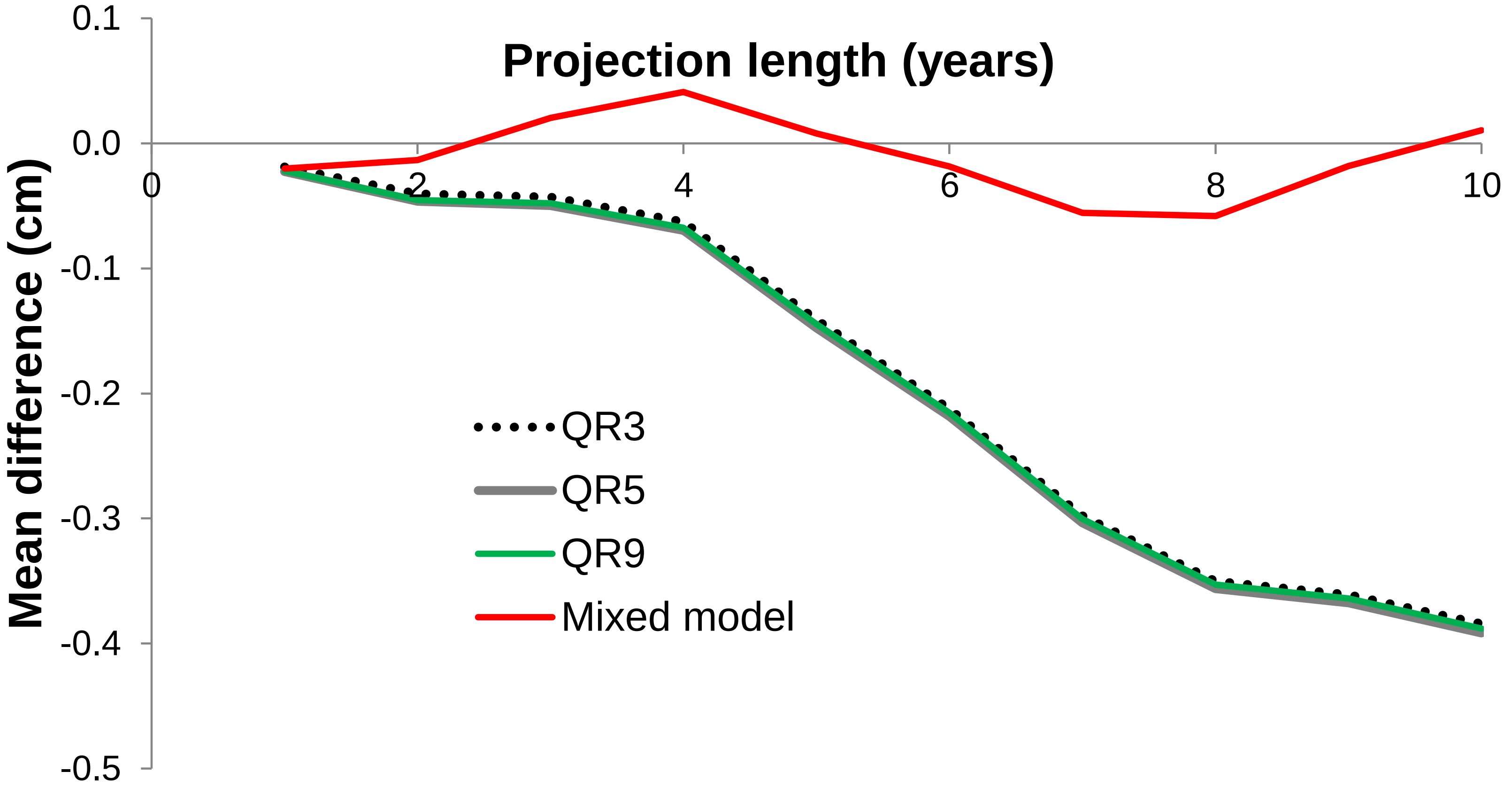
Mean abs. difference (cm)
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
Projection length (years)
QR3
QR5
QR9
Mixed model

Evaluation



1.0
0.9
0.8
0.7
Fit index (R^2)
0
2
4
6
8
10
Projection length (years)
(c)
QR3
QR5
QR9
Mixed model

Evaluation


Projection length (years)
Mean difference (cm)
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0
2
4
6
8
10
QR3
QR5
QR9
Mixed model

Postscript

Parameters of $f(x)$ are obtained by minimizing

$$S = \sum_{y_i \geq \hat{y}_i} \tau |y_i - \hat{y}_i| + \sum_{y_i < \hat{y}_i} (1 - \tau) |y_i - \hat{y}_i|$$

Postscript

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$$S = \sum_i \sum_j I_\tau \, |y_{ij} - \hat{y}_{ij}|$$

Postscript

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$$S = \sum_i \sum_j I_\tau \, |y_{ij} - \hat{y}_{ij}|$$

$$S = \sum_i \sum_j w_i \, I_\tau |y_{ij} - \hat{y}_{ij}|$$

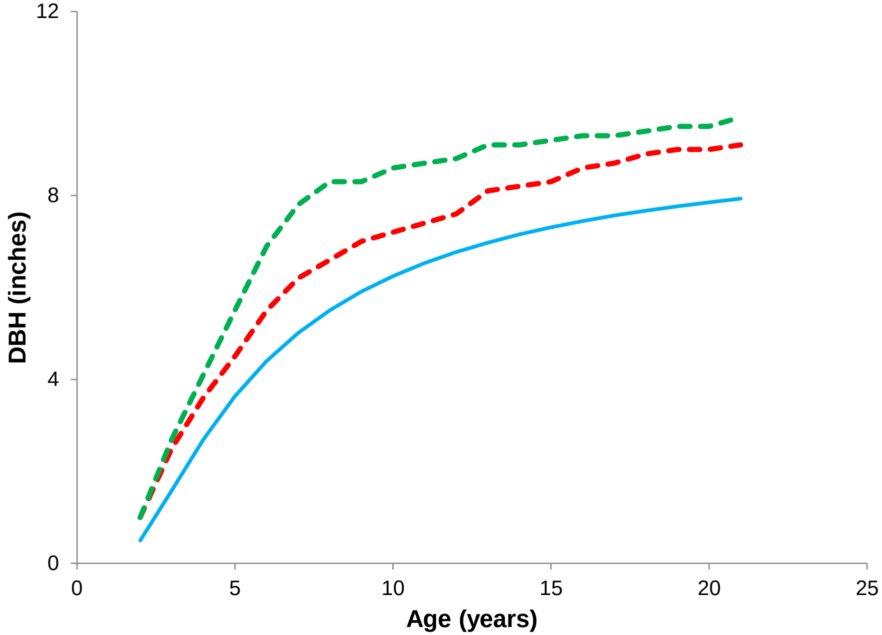
Postscript

Parameters of $f(x)$ are obtained by minimizing

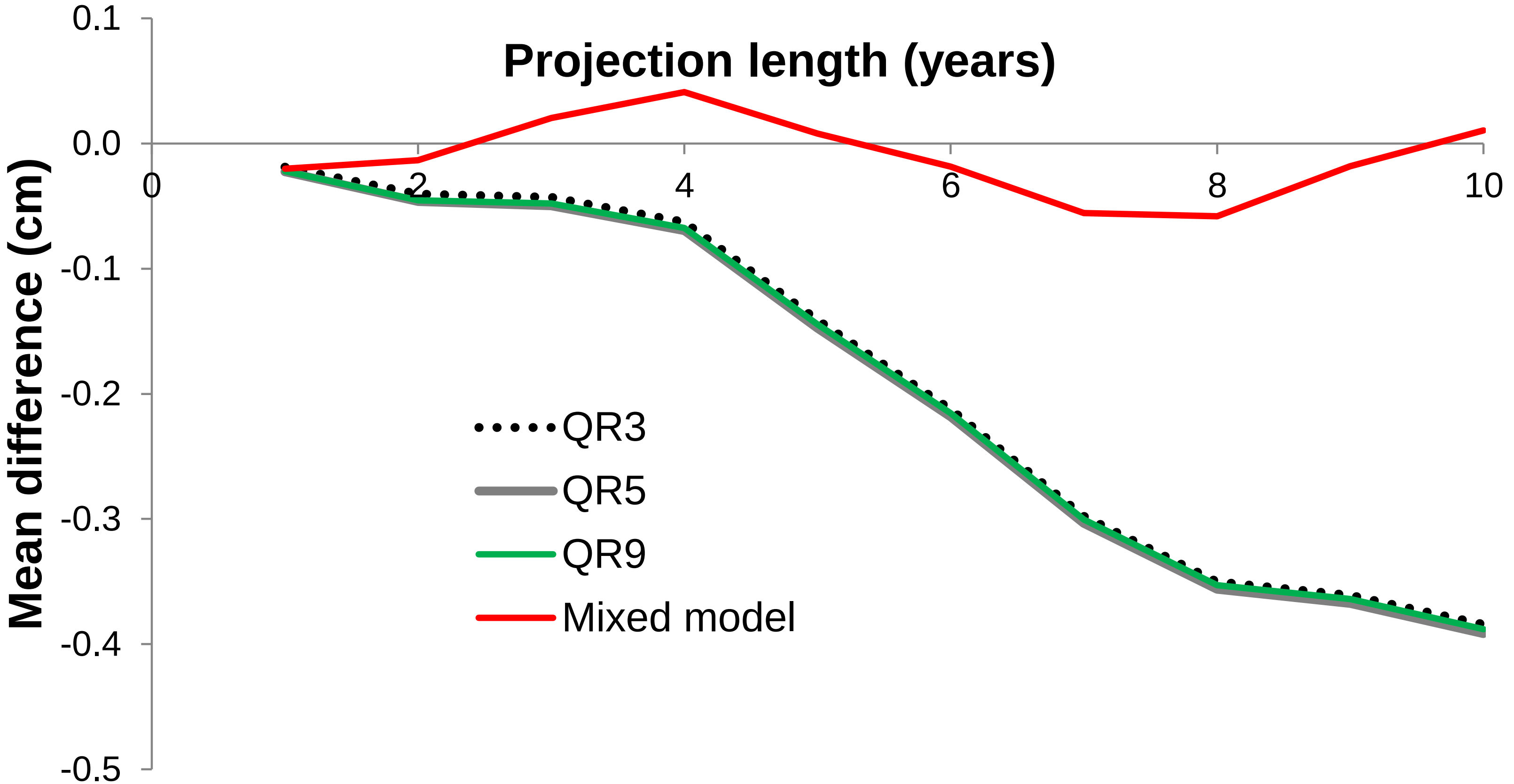
$$S = \sum_i \sum_j w_i \, I_\tau |y_{ij} - \hat{y}_{ij}|$$

$$w_i = \frac{1}{u_i}$$

$$u_i = \frac{1}{n} \sum_j |\varepsilon_{ij} - \bar{\varepsilon}_i|$$



Evaluation



Projection length (years)
Mean difference (cm)
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0
2
4
6
8
10
QR3
QR5
QR9
Mixed model